ICSE Class 10 Mathematics

Chapter-wise Step-by-Step Practice Solutions

Based on the Latest Exam Pattern and Syllabus

Chapter 1: Goods and Services Tax (GST)

Question 1: A dealer in Mumbai buys an article for Rs. 6000 and sells it to a consumer in Mumbai at a profit of 15%. If the rate of GST is 12%, find:

(i) The selling price of the article (excluding tax).

(ii) The CGST and SGST paid by the consumer.

(iii) The total amount paid by the consumer.

Solution:

Given:

- Cost Price (CP) for dealer = Rs. 6000
- Profit percentage = 15%
- Rate of GST = 12%

Step-by-step Calculation:

(i) **Selling Price (excluding tax):**

$$\begin{aligned}\text{Profit} &= 15\% \text{ of CP} \\ &= \frac{15}{100} \times 6000 = \text{Rs. } 900 \\ \text{Selling Price (SP)} &= \text{CP} + \text{Profit} \\ &= 6000 + 900 = \text{Rs. } 6900\end{aligned}$$

(ii) **CGST and SGST paid by the consumer:** Since the transaction is intra-state (Mumbai to Mumbai), the GST is divided equally into CGST and SGST.

$$\text{Rate of CGST} = \text{Rate of SGST} = \frac{12\%}{2} = 6\%$$

$$\text{CGST} = 6\% \text{ of SP} = \frac{6}{100} \times 6900 = \text{Rs. } 414$$

$$\text{SGST} = 6\% \text{ of SP} = \frac{6}{100} \times 6900 = \text{Rs. } 414$$

(iii) **Total amount paid by the consumer:**

$$\begin{aligned}\text{Total Amount} &= \text{SP} + \text{CGST} + \text{SGST} \\ &= 6900 + 414 + 414 = \text{Rs. } 7728\end{aligned}$$

Final Answer: (i) Rs. 6900, (ii) CGST = Rs. 414, SGST = Rs. 414, (iii) Rs. 7728.

Chapter 2: Banking

Question 2: Katrina opened a Recurring Deposit Account of Rs. 600 per month for 43 months. If the bank pays interest at the rate of 8% per annum, find the maturity value of her account.

Solution:

Given:

- Monthly Installment (P) = Rs. 600
- Number of months $(n) = 43$
- Rate of Interest $(r) = 8\%$ p.a.

Step-by-step Calculation:

$$\begin{aligned}\text{Equivalent Principal for 1 month } (P_{eq}) &= P \times \frac{n(n+1)}{2} \\ &= 600 \times \frac{43 \times 44}{2} \\ &= 600 \times 946 = \text{Rs. } 567,600\end{aligned}$$

Now, calculating the Interest (I):

$$\begin{aligned}I &= \frac{P_{eq} \times r \times 1}{100 \times 12} \\ &= \frac{567600 \times 8 \times 1}{100 \times 12} \\ &= \frac{5676 \times 8}{12} \\ &= 473 \times 8 = \text{Rs. } 3784\end{aligned}$$

Calculating the Maturity Value (MV):

$$\begin{aligned}\text{Total Principal Deposited} &= P \times n = 600 \times 43 = \text{Rs. } 25,800 \\ \text{Maturity Value (MV)} &= \text{Total Principal Deposited} + \text{Interest} \\ &= 25800 + 3784 = \text{Rs. } 29,584\end{aligned}$$

Final Answer: The maturity value of the account is Rs. 29,584.

Chapter 3: Linear Inequations

Question 3: Solve the following inequation and represent the solution set on a number line:

$$-3 < x - 2 \leq 9 - 2x, \quad x \in \mathbb{N}$$

Solution:

Step-by-step Calculation: We split the inequation into two parts and solve them simultaneously:

Part 1:

$$\begin{aligned} -3 &< x - 2 \\ -3 + 2 &< x \\ -1 &< x \quad \text{or} \quad x > -1 \end{aligned}$$

Part 2:

$$\begin{aligned} x - 2 &\leq 9 - 2x \\ x + 2x &\leq 9 + 2 \\ 3x &\leq 11 \\ x &\leq \frac{11}{3} \\ x &\leq 3.66... \end{aligned}$$

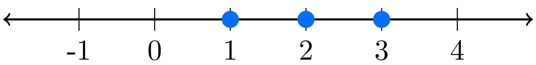
Combining Part 1 and Part 2:

$$-1 < x \leq 3.66...$$

Since it is given that $x \in \mathbb{N}$ (Natural Numbers, i.e., $\{1, 2, 3, 4, ...\}$), the values of x that satisfy the combined condition are:

$$x \in \{1, 2, 3\}$$

Number Line Representation:



Final Answer: Solution set $= \{1, 2, 3\}$.

Chapter 4: Quadratic Equations

Question 4: Solve the quadratic equation $2x^2 - 5x - 4 = 0$ using the quadratic formula. Give your answer correct to two decimal places.

Solution:

Given Equation: $2x^2 - 5x - 4 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$, we have: $a = 2, \quad b = -5, \quad c = -4$

Using the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-4)}}{2(2)}$$
$$x = \frac{5 \pm \sqrt{25 + 32}}{4}$$
$$x = \frac{5 \pm \sqrt{57}}{4}$$

Finding the square root of 57: $\sqrt{57} \approx 7.549...$ (Taking 7.55 for calculation)

$$\text{Case 1: } x = \frac{5 + 7.55}{4} = \frac{12.55}{4} = 3.1375$$
$$\text{Case 2: } x = \frac{5 - 7.55}{4} = \frac{-2.55}{4} = -0.6375$$

Rounding off to two decimal places: $x_1 \approx 3.14$ and $x_2 \approx -0.64$
Final Answer: $x = 3.14$ or $x = -0.64$.

Chapter 5: Matrices

Question 5: If $A = \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix}$, find the matrix $A^2 - 2A$.

Solution:

Step 1: Calculate A^2

$$\begin{aligned} A^2 = A \times A &= \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix} \\ &= \begin{pmatrix} (2)(2) + (1)(0) & (2)(1) + (1)(-2) \\ (0)(2) + (-2)(0) & (0)(1) + (-2)(-2) \end{pmatrix} \\ &= \begin{pmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{pmatrix} \\ &= \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \end{aligned}$$

Step 2: Calculate $2A$

$$2A = 2\begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & -4 \end{pmatrix}$$

Step 3: Calculate $A^2 - 2A$

$$\begin{aligned} A^2 - 2A &= \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 0 & -4 \end{pmatrix} \\ &= \begin{pmatrix} 4-4 & 0-2 \\ 0-0 & 4-(-4) \end{pmatrix} \\ &= \begin{pmatrix} 0 & -2 \\ 0 & 8 \end{pmatrix} \end{aligned}$$

Final Answer: $A^2 - 2A = \begin{pmatrix} 0 & -2 \\ 0 & 8 \end{pmatrix}$.

Chapter 6: Trigonometric Identities

Question 6: Prove the following identity:

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

Solution:

Proof: Taking the Left Hand Side (LHS):

$$\text{LHS} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

Rationalizing the denominator inside the square root by multiplying numerator and denominator by $(1+\sin A)$:

$$\begin{aligned}\text{LHS} &= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}} \\ &= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}\end{aligned}$$

Using the fundamental identity $\sin^2 A + \cos^2 A = 1$, we know that $1-\sin^2 A = \cos^2 A$:

$$\begin{aligned}\text{LHS} &= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} \\ &= \frac{1+\sin A}{\cos A}\end{aligned}$$

Separating the terms in the numerator:

$$\text{LHS} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

Using standard trigonometric ratios ($\frac{1}{\cos A} = \sec A$ and $\frac{\sin A}{\cos A} = \tan A$):

$$\begin{aligned}\text{LHS} &= \sec A + \tan A \\ &= \text{RHS}\end{aligned}$$

Hence Proved.