Class 10th Mathematics
Chapter 1: Real Numbers

Extra Practice Questions & Detailed Solutions (Based on Latest Exam Pattern)

Part A: Practice Questions

Section I: Very Short Answer Type Questions (1 Mark Each)

Q1. Find the HCF of 96 and 404 by the prime factorization method.

Q2. Given that $\text{HCF}(306, 657) = 9$, find the $\text{LCM}(306, 657)$.

Q3. State the Fundamental Theorem of Arithmetic.

Q4. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Section II: Short Answer Type Questions (2-3 Marks Each)

Q5. Check whether 6^n can end with the digit 0 for any natural number n.

Q6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Q7. Find the largest number which divides 70 and 125, leaving remainders 5 and 8, respectively.

Q8. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at 12 noon, at what time will they beep again for the first time?

Section III: Long Answer Type Questions (4-5 Marks Each)

Q9. Prove that $\sqrt{3}$ is an irrational number.

Q10. Prove that $5 - 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is irrational.

Q11. In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm, and 90 cm, respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Part B: Detailed Step-by-Step Solutions

Sol 1. HCF of 96 and 404:

First, we find the prime factorization of both numbers:
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$
$404 = 2 \times 2 \times 101 = 2^2 \times 101$
The HCF is the product of the smallest power of each common prime factor.
Common prime factor is 2. The smallest power is 2^2.
$\text{HCF}(96, 404) = 2^2 = 4$.

Sol 2. Finding LCM given HCF:

We know the relationship between HCF, LCM, and the product of two numbers:
$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$
Given: $a = 306$, $b = 657$, and $\text{HCF} = 9$.
$9 \times \text{LCM} = 306 \times 657$
$\text{LCM} = \frac{306 \times 657}{9}$
$\text{LCM} = 34 \times 657 = 22338$.

Sol 3. Fundamental Theorem of Arithmetic:

Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Sol 4. Can HCF be 18 and LCM be 380?

No. The HCF of two numbers must always be a factor of their LCM.
Let's divide 380 by 18:
$380 \div 18 = 21.11$ (It leaves a remainder of 2).
Since 18 does not exactly divide 380, two numbers cannot have 18 as their HCF and 380 as their LCM.

Sol 5. Checking if 6^n ends with 0:

If any number ends with the digit 0, it must be divisible by 10. Therefore, its prime factorization must contain at least one pair of 2 and 5.
Prime factorization of $6^n = (2 \times 3)^n = 2^n \times 3^n$.
We can observe that the prime factorization of 6^n does not contain the prime number 5.
By the uniqueness of the Fundamental Theorem of Arithmetic, there are no other primes in the factorization of 6^n.
Hence, for any natural number n, 6^n can never end with the digit 0.

Sol 6. Why the given numbers are composite:

Case 1:
Expression: $7 \times 11 \times 13 + 13$
Take 13 as a common factor:
$= 13 \times (7 \times 11 + 1)$
$= 13 \times (77 + 1) = 13 \times 78 = 13 \times 13 \times 2 \times 3$
Since the expression has factors other than 1 and itself, it is a composite number.

Case 2:
Expression: $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
Take 5 as a common factor:
$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$
$= 5 \times (1008 + 1) = 5 \times 1009$
Since it can be expressed as a product of primes (5 and 1009), it is a composite number.

Sol 7. Largest number dividing 70 and 125 with remainders 5 and 8:
The required number will exactly divide $(70-5)$ and $(125-8)$.
$70-5=65$
$125-8=117$
We need to find the HCF of 65 and 117.
Prime factorization:
$65=5\times 13$
$117=3\times 3\times 13=3^2\times 13$
The common factor is 13.
$\text{HCF}(65,117)=13$.
Therefore, the largest number is 13.

Sol 8. Alarm clocks ringing together:
The time after which the clocks will beep together again is the LCM of their intervals.
Intervals: 50 seconds and 48 seconds.
$50=2\times 5^2$
$48=2^4\times 3$
$\text{LCM}(50,48)=2^4\times 3\times 5^2=16\times 3\times 25=1200$ seconds.
Convert seconds to minutes: $\frac{1200}{60}=20$ minutes.
If they beeped together at 12:00 noon, they will beep together again at **12:20 PM**.

Sol 9. Prove that $\sqrt{3}$ is irrational:
Let us assume, to the contrary, that $\sqrt{3}$ is a rational number.
Then, there exist co-prime integers a and b $(b\neq 0)$ such that:
$\sqrt{3}=\frac{a}{b}$
Squaring both sides, we get:
$3=\frac{a^2}{b^2}$
$\implies a^2=3b^2 \quad$ — (Equation 1)
This shows that a^2 is divisible by 3. By theorem, if a prime divides a^2, it also divides a. Therefore, a is divisible by 3.
Let $a=3c$ for some integer c.
Substituting $a=3c$ in Equation 1:
$(3c)^2=3b^2$
$9c^2=3b^2$
$\implies b^2=3c^2$
This means b^2 is divisible by 3, which implies b is also divisible by 3.
Therefore, a and b have at least 3 as a common factor.
But this contradicts the fact that a and b are co-prime.
This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.
Hence, we conclude that $\sqrt{3}$ is irrational.

Sol 10. Prove that $5-2\sqrt{3}$ is irrational:
Let us assume, to the contrary, that $5-2\sqrt{3}$ is rational.
Then, it can be written in the form $\frac{a}{b}$, where a and b are co-prime integers and

$b \neq 0$.
$5 - 2\sqrt{3} = \frac{a}{b}$
Rearranging the terms, we get:
$2\sqrt{3} = 5 - \frac{a}{b}$
$2\sqrt{3} = \frac{5b-a}{b}$
$\sqrt{3} = \frac{5b-a}{2b}$
Since a and b are integers, $5b - a$ and $2b$ are also integers. Therefore, $\frac{5b-a}{2b}$ is a rational number.
This implies that $\sqrt{3}$ is a rational number.
However, this contradicts the established fact that $\sqrt{3}$ is irrational.
This contradiction arose due to our false assumption.
Hence, $5 - 2\sqrt{3}$ is an irrational number.

Sol 11. Minimum distance for complete steps:
The minimum distance each should walk must be a multiple of the step length of each person. Therefore, we need to find the LCM of 80 cm, 85 cm, and 90 cm.
Prime factorizations:
$80 = 2^4 \times 5$
$85 = 5 \times 17$
$90 = 2 \times 3^2 \times 5$
To find the LCM, we take the highest power of each prime factor involved:
$\text{LCM}(80, 85, 90) = 2^4 \times 3^2 \times 5 \times 17$
$\text{LCM} = 16 \times 9 \times 5 \times 17$
$\text{LCM} = 144 \times 85 = 12240$ cm.
Converting to meters: 12240 cm = 122 m 40 cm.
The minimum distance each should walk is **12240 cm**.