Class 10th Mathematics

Chapter: Quadratic Equations

Comprehensive Practice Questions & Solutions

General Instructions:

- This practice paper is designed based on the latest Class 10 syllabus pattern.
- Section A contains Multiple Choice Questions (1 mark each).
- Section B contains Short Answer Type I questions (2 marks each).
- Section C contains Short Answer Type II questions (3 marks each).
- Section D contains Long Answer Type questions (4/5 marks each).
- Detailed step-by-step solutions are provided at the end of the document.

Section A: Multiple Choice Questions (1 Mark Each)

1. Which of the following is a quadratic equation?

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$

(b) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

(c) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k = -1$

(d) $x^3 - x^2 = (x - 1)^3$

2. If one root of the quadratic equation $2x^2 + kx - 6 = 0$ is 2, the value of k is:

(a) 1

(b) -1

(c) 2

(d) -2

3. The roots of the quadratic equation $x^2 - 0.04 = 0$ are:

(a) ± 0.2

(b) ± 0.02

(c) 0.4

(d) 2

4. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has:

(a) Two distinct real roots

(b) Two equal real roots

(c) No real roots

(d) More than 2 real roots

Section B: Short Answer Type I (2 Marks Each)

5. Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

6. Solve the quadratic equation by factorization: $x^2 - 3x - 10 = 0$.

7. Find the value(s) of k for which the quadratic equation $kx(x-2) + 6 = 0$ has two equal real roots.

8. Check whether the following is a quadratic equation: $(x-2)(x+1) = (x-1)(x+3)$.

Section C: Short Answer Type II (3 Marks Each)

9. Find the roots of the following quadratic equation using the quadratic formula: $3x^2 - 5x + 2 = 0$.

10. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

11. Find two consecutive positive integers, sum of whose squares is 365.

12. Solve for x: $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, where $x \neq -4, 7$.

Section D: Long Answer Type (5 Marks Each)

13. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

14. A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

15. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Detailed Solutions & Answer Key

Section A Solutions

1. **(d)** $x^3 - x^2 = (x-1)^3 \Rightarrow x^3 - x^2 = x^3 - 3x^2 + 3x - 1 \Rightarrow 2x^2 - 3x + 1 = 0$, which is a quadratic equation.
2. **(b)** Since 2 is a root, substitute $x = 2$: $2(2)^2 + k(2) - 6 = 0 \Rightarrow 8 + 2k - 6 = 0 \Rightarrow 2k = -2 \Rightarrow k = -1$.
3. **(a)** $x^2 = 0.04 \Rightarrow x = \pm\sqrt{0.04} \Rightarrow x = \pm 0.2$.
4. **(c)** Discriminant $D = b^2 - 4ac = (-\sqrt{5})^2 - 4(2)(1) = 5 - 8 = -3$. Since $D < 0$, the equation has no real roots.

Section B Solutions

5. Comparing with $ax^2 + bx + c = 0$, $a = 2, b = -4, c = 3$.
   Discriminant $D = b^2 - 4ac = (-4)^2 - 4(2)(3) = 16 - 24 = -8$.
   Since $D < 0$, the given quadratic equation has **no real roots**.
6. $x^2 - 3x - 10 = 0$
   Splitting the middle term: $x^2 - 5x + 2x - 10 = 0$
   $x(x-5) + 2(x-5) = 0 \Rightarrow (x-5)(x+2) = 0$
   Therefore, the roots are $x = 5$ and $x = -2$.
7. Equation: $kx^2 - 2kx + 6 = 0$. Here $a = k, b = -2k, c = 6$.
   For equal roots, $D = 0 \Rightarrow b^2 - 4ac = 0$.
   $(-2k)^2 - 4(k)(6) = 0 \Rightarrow 4k^2 - 24k = 0 \Rightarrow 4k(k-6) = 0$.
   So, $k = 0$ or $k = 6$. However, if $k = 0$, the equation is not quadratic. Therefore, $k = 6$.
8. LHS: $x^2 + x - 2x - 2 = x^2 - x - 2$
   RHS: $x^2 + 3x - x - 3 = x^2 + 2x - 3$
   Equating LHS and RHS: $x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow -3x + 1 = 0$.
   Since the highest power of x is 1, it is **not a quadratic equation**.

Section C Solutions

9. $3x^2 - 5x + 2 = 0$. Here $a = 3, b = -5, c = 2$.
   $D = (-5)^2 - 4(3)(2) = 25 - 24 = 1$.
   Using quadratic formula: $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-5) \pm \sqrt{1}}{2(3)} = \frac{5 \pm 1}{6}$.
   $x = \frac{6}{6} = 1$ and $x = \frac{4}{6} = \frac{2}{3}$.
   Roots are 1 and $\frac{2}{3}$.
10. Let Rehman's present age be x years.
    Age 3 years ago $= x - 3$. Age 5 years from now $= x + 5$.
    According to the question: $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$.

$\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$.
Cross-multiplying: $6x + 6 = x^2 + 2x - 15 \Rightarrow x^2 - 4x - 21 = 0$.
$(x - 7)(x + 3) = 0 \Rightarrow x = 7$ or $x = -3$.
Age cannot be negative. Therefore, Rehman's present age is 7 **years**.

11. Let the consecutive positive integers be x and $x + 1$.
$x^2 + (x + 1)^2 = 365 \Rightarrow x^2 + x^2 + 2x + 1 = 365 \Rightarrow 2x^2 + 2x - 364 = 0$.
Dividing by 2: $x^2 + x - 182 = 0$.
$(x + 14)(x - 13) = 0$. Since x is positive, $x = 13$.
The integers are 13 and 14.

12. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$
$\frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30} \Rightarrow \frac{-11}{x^2-3x-28} = \frac{11}{30}$.
Dividing both sides by 11: $\frac{-1}{x^2-3x-28} = \frac{1}{30}$.
$-30 = x^2 - 3x - 28 \Rightarrow x^2 - 3x + 2 = 0$.
$(x - 2)(x - 1) = 0 \Rightarrow x = 2$ or $x = 1$.
Roots are $x = 1, 2$.

Section D Solutions

13. Let the original speed of the train be x km/h.
Time taken at original speed $= \frac{360}{x}$ hours.
Time taken at increased speed $= \frac{360}{x+5}$ hours.
According to the question: $\frac{360}{x} - \frac{360}{x+5} = 1$.
$360\left(\frac{x+5-x}{x(x+5)}\right) = 1 \Rightarrow \frac{360\times5}{x^2+5x} = 1 \Rightarrow x^2 + 5x - 1800 = 0$.
Factorizing: $x^2 + 45x - 40x - 1800 = 0 \Rightarrow x(x + 45) - 40(x + 45) = 0$.
$(x + 45)(x - 40) = 0$. Since speed cannot be negative, $x = 40$.
The speed of the train is 40 **km/h**.

14. Let the speed of the stream be x km/h.
Speed of boat upstream $= (18 - x)$ km/h.
Speed of boat downstream $= (18 + x)$ km/h.
Time taken upstream $= \frac{24}{18-x}$ hours. Time taken downstream $= \frac{24}{18+x}$ hours.
Given: $\frac{24}{18-x} - \frac{24}{18+x} = 1$.
$24\left(\frac{18+x-(18-x)}{(18-x)(18+x)}\right) = 1 \Rightarrow 24\left(\frac{2x}{324-x^2}\right) = 1$.
$48x = 324 - x^2 \Rightarrow x^2 + 48x - 324 = 0$.
Factorizing: $x^2 + 54x - 6x - 324 = 0 \Rightarrow x(x + 54) - 6(x + 54) = 0$.
$(x + 54)(x - 6) = 0$. Since stream speed cannot be negative, $x = 6$.
The speed of the stream is 6 **km/h**.

15. Let the shorter side of the rectangular field be x metres.
Then, the longer side $= (x + 30)$ m.
The diagonal $= (x + 60)$ m.
By Pythagoras Theorem: $(x)^2 + (x + 30)^2 = (x + 60)^2$.
$x^2 + x^2 + 60x + 900 = x^2 + 120x + 3600$.
$x^2 - 60x - 2700 = 0$.
Factorizing: $x^2 - 90x + 30x - 2700 = 0 \Rightarrow x(x - 90) + 30(x - 90) = 0$.
$(x - 90)(x + 30) = 0$. Since length cannot be negative, $x = 90$.

Shorter side $= 90$ **m**.
Longer side $= 90 + 30 = 120$ **m**.