# Class 10 Maths NCERT Exemplar

## Comprehensive Chapter-wise Solutions

*Detailed, step-by-step practice solutions based on the latest exam pattern.*

## Chapter 1: Real Numbers

**Q. 1** Explain why $3 \times 5 \times 7 + 7$ is a composite number. ***Solution:***

Given the expression: $3 \times 5 \times 7 + 7$.
We can factor out the common term 7 from both parts of the expression:

$$\begin{aligned} 3 \times 5 \times 7 + 7 &= 7 \times (3 \times 5 + 1) \\ &= 7 \times (15 + 1) \\ &= 7 \times 16 \\ &= 7 \times 2^4 \end{aligned}$$

Since the given number can be expressed as a product of prime factors (i.e., it has factors other than 1 and itself), it is a composite number by definition.

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**Q. 2** Prove that $\sqrt{2}$ is an irrational number. ***Solution:***

We will prove this by the method of contradiction.
Let us assume, to the contrary, that $\sqrt{2}$ is a rational number.
Then, there exist co-prime integers $a$ and $b$ ($b \neq 0$) such that:

$$\sqrt{2} = \frac{a}{b}$$

Squaring both sides, we get:

$$2 = \frac{a^2}{b^2} \implies a^2 = 2b^2 \quad \text{— (i)}$$

Equation (i) shows that $a^2$ is divisible by 2. By the fundamental theorem of arithmetic, if a prime number divides $a^2$, it must also divide $a$.
So, $a$ is divisible by 2. Let $a = 2c$ for some integer $c$.
Substitute $a = 2c$ in equation (i):

$$\begin{aligned} (2c)^2 &= 2b^2 \\ 4c^2 &= 2b^2 \\ b^2 &= 2c^2 \quad \text{— (ii)} \end{aligned}$$

Equation (ii) shows that $b^2$ is divisible by 2, which means $b$ is also divisible by 2.
Therefore, both $a$ and $b$ share a common factor of 2. This contradicts our initial assumption that $a$ and $b$ are co-prime (having no common factor other than 1).
This contradiction arises because of our incorrect assumption that $\sqrt{2}$ is rational.
Hence, $\sqrt{2}$ is irrational.

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## Chapter 2: Polynomials

**Q. 3** If one zero of the polynomial $(a^2+9)x^2+13x+6a$ is reciprocal of the other, find the value of $a$. ***Solution:***

Let the zeroes of the given quadratic polynomial $P(x)=(a^2+9)x^2+13x+6a$ be $\alpha$ and $\frac{1}{\alpha}$.
We know that for a quadratic polynomial $Ax^2+Bx+C$, the product of the zeroes is given by $\frac{C}{A}$.
Here, $A=a^2+9$, $B=13$, and $C=6a$.
Product of zeroes:

$$\begin{aligned} \alpha \times \frac{1}{\alpha} &= \frac{6a}{a^2+9} \\ 1 &= \frac{6a}{a^2+9} \\ a^2+9 &= 6a \\ a^2-6a+9 &= 0 \end{aligned}$$

This is a perfect square equation:

$$\begin{aligned} (a-3)^2 &= 0 \\ a-3 &= 0 \implies a=3 \end{aligned}$$

Therefore, the value of $a$ is 3.

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**Q. 4** Find the zeroes of the quadratic polynomial $x^2+7x+10$, and verify the relationship between the zeroes and the coefficients. ***Solution:***

Let $P(x)=x^2+7x+10$.
To find the zeroes, we set $P(x)=0$ and factorize by splitting the middle term:

$$\begin{aligned} x^2+5x+2x+10 &= 0 \\ x(x+5)+2(x+5) &= 0 \\ (x+2)(x+5) &= 0 \end{aligned}$$

So, the zeroes are $\alpha=-2$ and $\beta=-5$.
**Verification:**
Sum of zeroes $(\alpha+\beta)=-2+(-5)=-7$.
From coefficients, Sum $=-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=-\frac{7}{1}=-7$. (Verified)
Product of zeroes $(\alpha\beta)=(-2)\times(-5)=10$.
From coefficients, Product $=\frac{\text{Constant term}}{\text{Coefficient of } x^2}=\frac{10}{1}=10$. (Verified)

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## Chapter 4: Quadratic Equations

**Q. 5** Find the values of $k$ for which the quadratic equation $kx(x-2)+6=0$ has two equal roots. ***Solution:***

First, write the equation in the standard form $ax^2+bx+c=0$:

$$kx^2-2kx+6=0$$

Here, $a = k$, $b = -2k$, and $c = 6$.
For a quadratic equation to have two equal real roots, its discriminant ($D$) must be equal to zero.

$$\begin{aligned} D = b^2 - 4ac &= 0 \\ (-2k)^2 - 4(k)(6) &= 0 \\ 4k^2 - 24k &= 0 \\ 4k(k-6) &= 0 \end{aligned}$$

This gives $k = 0$ or $k = 6$.
However, if $k = 0$, the original equation becomes $6 = 0$, which is not possible (it ceases to be a quadratic equation).
Therefore, $k = 6$.

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## Chapter 5: Arithmetic Progressions

**Q. 6** The sum of the first $n$ terms of an AP is given by $S_n = 4n^2 + 2n$. Find the $n$th term of this AP. ***Solution:***

We are given $S_n = 4n^2 + 2n$.
The $n$th term of an AP can be found using the formula: $a_n = S_n - S_{n-1}$.
First, find $S_{n-1}$ by replacing $n$ with $(n-1)$ in the given formula:

$$\begin{aligned} S_{n-1} &= 4(n-1)^2 + 2(n-1) \\ &= 4(n^2 - 2n + 1) + 2n - 2 \\ &= 4n^2 - 8n + 4 + 2n - 2 \\ &= 4n^2 - 6n + 2 \end{aligned}$$

Now, substitute $S_n$ and $S_{n-1}$ into the formula for $a_n$:

$$\begin{aligned} a_n &= (4n^2 + 2n) - (4n^2 - 6n + 2) \\ &= 4n^2 + 2n - 4n^2 + 6n - 2 \\ &= 8n - 2 \end{aligned}$$

Therefore, the $n$th term of the AP is $8n - 2$.

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## Chapter 6: Triangles

**Q. 7** In an equilateral triangle $ABC$, $D$ is a point on side $BC$ such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$. ***Solution:***

Let $ABC$ be an equilateral triangle with side length $a$. Thus, $AB = BC = CA = a$.
Given that $D$ is on $BC$ such that $BD = \frac{a}{3}$.
Draw an altitude $AE \perp BC$. In an equilateral triangle, the altitude bisects the base.
Therefore, $BE = EC = \frac{a}{2}$.
Now, calculate the length of $DE$:

$$DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{3a - 2a}{6} = \frac{a}{6}$$

In right-angled triangle $ABE$, apply Pythagoras theorem to find $AE^2$:

$$AE^2 = AB^2 - BE^2$$
$$AE^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

Now, in right-angled triangle $ADE$, apply Pythagoras theorem:

$$AD^2 = AE^2 + DE^2$$
$$AD^2 = \frac{3a^2}{4} + \left(\frac{a}{6}\right)^2$$
$$AD^2 = \frac{3a^2}{4} + \frac{a^2}{36}$$

Take the LCM of 4 and 36, which is 36:

$$AD^2 = \frac{27a^2 + a^2}{36}$$
$$AD^2 = \frac{28a^2}{36} = \frac{7a^2}{9}$$

Cross-multiplying gives:

$$9AD^2 = 7a^2$$

Since $a = AB$, we substitute it back:

$$9AD^2 = 7AB^2$$

Hence Proved.

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## Chapter 8: Introduction to Trigonometry

**Q. 8** Prove that: $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ ***Solution:***

Consider the Left Hand Side (LHS):

$$\text{LHS} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

Divide the numerator and the denominator by $\cos\theta$:

$$\begin{aligned}\text{LHS} &= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}} \\ &= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta} \\ &= \frac{(\tan\theta + \sec\theta) - 1}{\tan\theta - \sec\theta + 1}\end{aligned}$$

We know the trigonometric identity: $\sec^2\theta - \tan^2\theta = 1$. Substitute this for the '1' in the numerator:

$$\begin{aligned}\text{LHS} &= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1} \\ &= \frac{(\sec\theta + \tan\theta) - [(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)]}{\tan\theta - \sec\theta + 1}\end{aligned}$$

Factor out $(\sec\theta + \tan\theta)$ from the numerator:

$$\begin{aligned}\text{LHS} &= \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1} \\ &= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}\end{aligned}$$

Cancel the common term $(1 - \sec\theta + \tan\theta)$ from numerator and denominator:

$$\text{LHS} = \sec\theta + \tan\theta$$

Now, multiply and divide by $(\sec\theta - \tan\theta)$:

$$\begin{aligned}\text{LHS} &= \frac{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\sec\theta - \tan\theta} \\ &= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} \\ &= \frac{1}{\sec\theta - \tan\theta} = \text{RHS}\end{aligned}$$

Hence Proved.