Class 10 Maths Chapter 9: Some Applications of Trigonometry

Important Practice Questions & Detailed Solutions

Key Concepts & Formulas

Before attempting the questions, review these essential concepts for solving problems related to Heights and Distances:

- **Line of Sight:** The line drawn from the eye of an observer to the point in the object viewed by the observer.
- **Angle of Elevation:** The angle formed by the line of sight with the horizontal when the object is above the horizontal level (i.e., when we raise our head to look at the object).
- **Angle of Depression:** The angle formed by the line of sight with the horizontal when the object is below the horizontal level (i.e., when we lower our head to look at the object).
- **Trigonometric Ratios:** Remember the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for standard angles $(30°, 45°, 60°)$. In most height and distance problems, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$ is highly useful.

Section A: Multiple Choice Questions (1 Mark Each)

1. The height of a tower is 10 m. What is the length of its shadow when Sun's altitude is 45°?

(a) 10 m
(b) $10\sqrt{3}$ m
(c) $\frac{10}{\sqrt{3}}$ m
(d) 20 m

2. If the angle of elevation of a tower from a distance of 100 m from its foot is 60°, then the height of the tower is:

(a) $100\sqrt{3}$ m
(b) $\frac{100}{\sqrt{3}}$ m

(c) $50\sqrt{3}$ m
(d) $\frac{200}{\sqrt{3}}$ m

3. A ladder 15 m long makes an angle of 60° with the wall. Find the height of the point where the ladder touches the wall.

(a) $15\sqrt{3}$ m
(b) 7.5 m
(c) $7.5\sqrt{3}$ m
(d) 15 m

4. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. The height of the tower is:

(a) $20(\sqrt{3}-1)$ m
(b) $20(\sqrt{3}+1)$ m
(c) $20\sqrt{3}$ m
(d) 40 m

5. The ratio of the length of a vertical rod and the length of its shadow is $1 : \sqrt{3}$. The angle of elevation of the sun is:

(a) 30°
(b) 45°
(c) 60°
(d) 90°

Section B: Short Answer Type Questions (2-3 Marks)

6. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

7. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

8. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

Section C: Long Answer Type Questions (4-5 Marks)

9. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storied building are $30°$ and $45°$, respectively. Find the height of the multi-storied building and the distance between the two buildings.

10. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of $30°$, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be $60°$. Find the time taken by the car to reach the foot of the tower from this point.

11. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is $60°$ and the angle of depression of its foot is $45°$. Determine the height of the tower.

Section D: Case Study Based Question (4 Marks)

12. **Aviation Observation:** An observer on the ground spots an airplane flying at a constant height of $3000\sqrt{3}$ meters. At a given instant, the angle of elevation of the airplane from the observer's eye is $60°$. After a flight of 15 seconds, the angle of elevation changes to $30°$.

(i) Draw a neat labeled diagram to represent the above situation. (1 Mark)

(ii) Find the horizontal distance covered by the airplane in 15 seconds. (2 Marks)

(iii) Calculate the speed of the airplane in km/hr. (1 Mark)

Detailed Solutions & Answers

Section A: Multiple Choice Questions

1. **Answer: (a) 10 m**
   Explanation: Let the shadow be x. $\tan 45^\circ = \frac{\text{Height}}{\text{Shadow}} \Rightarrow 1 = \frac{10}{x} \Rightarrow x = 10$ m.

2. **Answer: (a) $100\sqrt{3}$ m**
   Explanation: $\tan 60^\circ = \frac{h}{100} \Rightarrow \sqrt{3} = \frac{h}{100} \Rightarrow h = 100\sqrt{3}$ m.

3. **Answer: (b) 7.5 m**
   Explanation: Let the height be h. The angle with the *wall* is 60°, so the angle with the ground is 30°. Or, $\cos 60^\circ = \frac{h}{15} \Rightarrow \frac{1}{2} = \frac{h}{15} \Rightarrow h = 7.5$ m.

4. **Answer: (a) $20(\sqrt{3} - 1)$ m**
   Explanation: Let building height be $AB = 20$m, tower be $BC = h$. Base distance $OA = x$. From $\triangle OAB$, $\tan 45^\circ = \frac{20}{x} \Rightarrow x = 20$ m. From $\triangle OAC$, $\tan 60^\circ = \frac{20+h}{x} \Rightarrow \sqrt{3} = \frac{20+h}{20} \Rightarrow 20\sqrt{3} = 20 + h \Rightarrow h = 20(\sqrt{3} - 1)$ m.

5. **Answer: (a) 30°**
   Explanation: $\tan\theta = \frac{\text{Height}}{\text{Shadow}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$.

Section B: Short Answer Type Questions

6. **Solution:**
   Let the length of the string be L. The height of the kite is $h = 60$ m. The angle of elevation $\theta = 60^\circ$.
   In the right-angled triangle formed, $\sin 60^\circ = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{60}{L}$
   $\frac{\sqrt{3}}{2} = \frac{60}{L} \Rightarrow L = \frac{120}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}$ m.
   Length of the string is $40\sqrt{3}$ m.

7. **Solution:**
   Let the height of the tower be h and the length of the shadow when the sun's altitude is 60° be x.
   When the altitude is 30°, the shadow is $(x + 40)$ m.
   In the first right triangle: $\tan 60^\circ = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)
   In the second right triangle: $\tan 30^\circ = \frac{h}{x+40} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+40} \Rightarrow x + 40 = h\sqrt{3}$ ...(ii)
   Substitute x from (i) into (ii):
   $\frac{h}{\sqrt{3}} + 40 = h\sqrt{3} \Rightarrow 40 = h\sqrt{3} - \frac{h}{\sqrt{3}} \Rightarrow 40 = \frac{3h-h}{\sqrt{3}} \Rightarrow 40 = \frac{2h}{\sqrt{3}}$
   $2h = 40\sqrt{3} \Rightarrow h = 20\sqrt{3}$ m.
   The height of the tower is $20\sqrt{3}$ m.

8. **Solution:**
   Let the height of each pole be h. Let the distance of the point from the first pole be x. Then the distance from the second pole is $(80 - x)$.
   For the first pole: $\tan 60^\circ = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ ...(i)
   For the second pole: $\tan 30^\circ = \frac{h}{80-x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{80-x} \Rightarrow h = \frac{80-x}{\sqrt{3}}$ ...(ii)
   Equating (i) and (ii):
   $x\sqrt{3} = \frac{80-x}{\sqrt{3}} \Rightarrow 3x = 80 - x \Rightarrow 4x = 80 \Rightarrow x = 20$ m.

Distance from second pole $= 80 - 20 = 60$ m.
Height $h = 20\sqrt{3}$ m.
Height of the poles is $20\sqrt{3}$ m. The point is 20 m and 60 m away from the poles.

Section C: Long Answer Type Questions

9. Solution:
Let the height of the multi-storied building be H and the distance between the buildings be x. The height of the smaller building is 8 m.
Draw a horizontal line from the top of the 8 m building to the multi-storied building. This divides H into two parts: $(H-8)$ and 8.
From the top of the multi-storied building:
Angle of depression to the bottom of the smaller building is 45°.
$\tan 45^\circ = \frac{H}{x} \Rightarrow 1 = \frac{H}{x} \Rightarrow H = x \quad ...(i)$
Angle of depression to the top of the smaller building is 30°.
$\tan 30^\circ = \frac{H-8}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{H-8}{x} \Rightarrow x = \sqrt{3}(H-8) \quad ...(ii)$
Since $x = H$, substitute in (ii):
$H = \sqrt{3}H - 8\sqrt{3} \Rightarrow H(\sqrt{3}-1) = 8\sqrt{3}$
$H = \frac{8\sqrt{3}}{\sqrt{3}-1}$. Rationalizing the denominator:
$H = \frac{8\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{8(3+\sqrt{3})}{2} = 4(3+\sqrt{3})$ m.
Height of the building is $4(3+\sqrt{3})$ m, and the distance between them is also $4(3+\sqrt{3})$ m.

10. Solution:
Let the height of the tower be h. Let the initial position of the car be A, position after 6 seconds be B, and the foot of the tower be C.
Angle of depression at A is 30°, so $\angle DAC = 30^\circ$ (where D is top of the tower). Thus, $\angle DAC = \angle BCA = 30^\circ$ (alternate interior angles).
Similarly, angle at B is 60°.
In $\triangle DCB$: $\tan 60^\circ = \frac{h}{BC} \Rightarrow \sqrt{3} = \frac{h}{BC} \Rightarrow BC = \frac{h}{\sqrt{3}}$
In $\triangle DCA$: $\tan 30^\circ = \frac{h}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AC} \Rightarrow AC = h\sqrt{3}$
Distance covered in 6 seconds $= AB = AC - BC = h\sqrt{3} - \frac{h}{\sqrt{3}} = \frac{3h-h}{\sqrt{3}} = \frac{2h}{\sqrt{3}}$.
Speed of the car $= \frac{\text{Distance}}{\text{Time}} = \frac{2h/\sqrt{3}}{6} = \frac{h}{3\sqrt{3}}$ m/s.
Time taken to cover remaining distance BC:
Time $= \frac{BC}{\text{Speed}} = \frac{h/\sqrt{3}}{h/3\sqrt{3}} = \frac{h}{\sqrt{3}} \times \frac{3\sqrt{3}}{h} = 3$ seconds.
The car will take 3 more seconds to reach the foot of the tower.

11. Solution:
Let the building be AB ($AB = 7$ m) and the cable tower be CD.
Draw AE parallel to BD, so AE intersects CD at E. Then $CE = h$ and $ED = 7$ m. The total height of the tower is $(h+7)$ m.
Let the distance between them be x ($BD = AE = x$).
Angle of elevation to the top (C) is 60°: In $\triangle CAE$, $\tan 60^\circ = \frac{CE}{AE} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \quad ...(i)$
Angle of depression to the foot (D) is 45°: In $\triangle EAD$, $\tan 45^\circ = \frac{ED}{AE} \Rightarrow 1 = \frac{7}{x} \Rightarrow x = 7$ m.

Substitute $x = 7$ in (i): $h = 7\sqrt{3}$ m.
Total height of the tower $= h + 7 = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$ m.
The height of the cable tower is $7(\sqrt{3} + 1)$ m.

Section D: Case Study Based Question

12. Solution:

(ii) Let the height of the airplane be $h = 3000\sqrt{3}$ m. Let the observer be at point O.
Let the initial position of the plane be A and the final position be B. Draw perpendiculars AC and BD to the ground.
In $\triangle OCA$: $\tan 60° = \frac{AC}{OC} \Rightarrow \sqrt{3} = \frac{3000\sqrt{3}}{OC} \Rightarrow OC = 3000$ m.
In $\triangle ODB$: $\tan 30° = \frac{BD}{OD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{OD} \Rightarrow OD = 3000\sqrt{3} \times \sqrt{3} = 9000$ m.
Horizontal distance covered $= CD = OD - OC = 9000 - 3000 = 6000$ m.
The horizontal distance covered is 6000 m.

(iii) Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{6000 \text{ m}}{15 \text{ s}} = 400$ m/s.
To convert to km/hr, multiply by $\frac{18}{5}$:
Speed $= 400 \times \frac{18}{5} = 80 \times 18 = 1440$ km/hr.
The speed of the airplane is 1440 km/hr.