CBSE Class 10 Maths Solutions: Comprehensive Practice Guide

Detailed step-by-step solutions based on the latest Class 10 NCERT curriculum.

Chapter 1: Real Numbers

Question 1: Prove that $\sqrt{2}$ is an irrational number.

Solution: Let us assume, to the contrary, that $\sqrt{2}$ is a rational number. Then, there exist co-prime integers a and b $(b \neq 0)$ such that:

$$\sqrt{2} = \frac{a}{b}$$

Squaring both sides, we get:

$$2 = \frac{a^2}{b^2} \implies a^2 = 2b^2 \quad \text{— (Equation 1)}$$

This shows that a^2 is divisible by 2. By theorem, if a prime number divides a^2, it also divides a. Therefore, a is divisible by 2. Let $a = 2c$ for some integer c. Substituting this into Equation 1:

$$(2c)^2 = 2b^2 \implies 4c^2 = 2b^2 \implies b^2 = 2c^2$$

This means b^2 is divisible by 2, and hence b is also divisible by 2. Since both a and b are divisible by 2, they have a common factor of 2. This contradicts our initial assumption that a and b are co-prime. Hence, our assumption is wrong, and $\sqrt{2}$ is irrational.

Question 2: Find the LCM and HCF of 26 and 91, and verify that LCM $\times$ HCF = product of the two numbers.

Solution: First, find the prime factorization of both numbers:

$$26 = 2 \times 13$$
$$91 = 7 \times 13$$

- **HCF (Highest Common Factor):** The product of the smallest power of each common prime factor. Here, the common prime factor is 13. Thus, HCF = 13.

- **LCM (Least Common Multiple):** The product of the greatest power of each prime factor involved. LCM $= 2 \times 7 \times 13 = 182$.

Verification:

$$\text{LCM} \times \text{HCF} = 182 \times 13 = 2366$$
$$\text{Product of numbers} = 26 \times 91 = 2366$$

Since LCM $\times$ HCF = Product of the numbers, the relationship is verified.

Chapter 2: Polynomials

Question 3: Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$, and verify the relationship between the zeroes and the coefficients.

Solution: Let $p(x) = x^2 - 2x - 8$. To find the zeroes, set $p(x) = 0$. We use the method of splitting the middle term:

$$x^2 - 4x + 2x - 8 = 0$$
$$x(x-4) + 2(x-4) = 0$$
$$(x-4)(x+2) = 0$$

So, the zeroes are $x = 4$ and $x = -2$. Let $\alpha = 4$ and $\beta = -2$.

Verification: Comparing $x^2 - 2x - 8$ with the standard form $ax^2 + bx + c$, we get $a = 1, b = -2, c = -8$.

$$\text{Sum of zeroes } (\alpha + \beta) = 4 + (-2) = 2$$
$$\text{Formula } \left(-\frac{b}{a}\right) = -\frac{-2}{1} = 2 \quad \text{(Verified)}$$

$$\text{Product of zeroes } (\alpha\beta) = 4 \times (-2) = -8$$
$$\text{Formula } \left(\frac{c}{a}\right) = \frac{-8}{1} = -8 \quad \text{(Verified)}$$

Chapter 5: Arithmetic Progressions

Question 4: The sum of the first 14 terms of an AP is 1050 and its first term is 10. Find the 20th term.

Solution: Given: First term, $a = 10$
Sum of first 14 terms, $S_{14} = 1050$
Number of terms, $n = 14$

The formula for the sum of n terms is:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substituting the given values:

$$\begin{aligned}
1050 &= \frac{14}{2}[2(10) + (14-1)d] \\
1050 &= 7[20 + 13d] \\
\frac{1050}{7} &= 20 + 13d \\
150 &= 20 + 13d \\
13d &= 130 \implies d = 10
\end{aligned}$$

Now, to find the 20th term (a_{20}):

$$\begin{aligned}
a_n &= a + (n-1)d \\
a_{20} &= 10 + (20-1)(10) \\
a_{20} &= 10 + 19 \times 10 \\
a_{20} &= 10 + 190 = 200
\end{aligned}$$

Therefore, the 20th term of the AP is 200.

Chapter 8: Introduction to Trigonometry

Question 5: In $\triangle ABC$, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine $\sin A$ and $\cos A$.

Solution: By Pythagoras theorem in $\triangle ABC$:

$$\begin{aligned}
AC^2 &= AB^2 + BC^2 \\
AC^2 &= (24)^2 + (7)^2 \\
AC^2 &= 576 + 49 = 625 \\
AC &= \sqrt{625} = 25 \text{ cm}
\end{aligned}$$

Now, evaluating the trigonometric ratios with respect to angle A:

- Perpendicular (opposite to A) = $BC = 7$ cm
- Base (adjacent to A) = $AB = 24$ cm
- Hypotenuse = $AC = 25$ cm

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

Question 6: Prove the following identity: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

Solution: Taking the Left Hand Side (LHS):

$$\text{LHS} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

Taking the LCM of the denominators:

$$\text{LHS} = \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)\cos A}$$

Expanding $(1 + \sin A)^2$:

$$\text{LHS} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1 + \sin A)\cos A}$$

We know the trigonometric identity $\sin^2 A + \cos^2 A = 1$. Substituting this:

$$\begin{aligned}\text{LHS} &= \frac{1 + 1 + 2\sin A}{(1 + \sin A)\cos A} \\ &= \frac{2 + 2\sin A}{(1 + \sin A)\cos A} \\ &= \frac{2(1 + \sin A)}{(1 + \sin A)\cos A}\end{aligned}$$

Canceling the common term $(1 + \sin A)$ from numerator and denominator:

$$\text{LHS} = \frac{2}{\cos A} = 2\sec A = \text{RHS}$$

Hence, Proved.