CBSE Class 10 Mathematics

Chapter 2: Polynomials

Comprehensive Study Material, Formulas, and Practice Questions

1 Introduction to Polynomials

An algebraic expression of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_n, a_{n-1}, \ldots, a_0$ are real numbers, $a_n \neq 0$, and n is a non-negative integer, is called a **polynomial** in variable x.

Degree of a Polynomial

The highest power of x in $p(x)$ is called the **degree** of the polynomial.

- **Linear Polynomial:** A polynomial of degree 1. Standard form: $ax + b$ (where $a \neq 0$).
- **Quadratic Polynomial:** A polynomial of degree 2. Standard form: $ax^2 + bx + c$ (where $a \neq 0$).
- **Cubic Polynomial:** A polynomial of degree 3. Standard form: $ax^3 + bx^2 + cx + d$ (where $a \neq 0$).

2 Geometrical Meaning of the Zeroes of a Polynomial

The **zeroes** (or roots) of a polynomial $p(x)$ are the x-coordinates of the points where the graph of $y = p(x)$ intersects the x-axis.

- A linear polynomial has exactly one zero.
- A quadratic polynomial can have at most two zeroes. Its graph is a curve called a *parabola* (opening upwards if $a > 0$ or downwards if $a < 0$).
- A polynomial of degree n has at most n zeroes.

3 Relationship Between Zeroes and Coefficients

3.1 For a Quadratic Polynomial

Let α and β be the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, where $a \neq 0$.

$$\textbf{Sum of zeroes} \quad (\alpha + \beta) = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$$

$$\textbf{Product of zeroes} \quad (\alpha\beta) = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$$

Forming a Quadratic Polynomial: If the zeroes α and β are given, the quadratic polynomial can be formed as:

$$p(x) = k\left[x^2 - (\alpha+\beta)x + \alpha\beta\right]$$

where k is any non-zero real constant.

3.2 For a Cubic Polynomial

Let α, β, and γ be the zeroes of the cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

4 Solved Examples

Example 1: Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Solution:

Let $p(x) = x^2 + 7x + 10$.

To find the zeroes, set $p(x) = 0$. By splitting the middle term:

$$x^2 + 5x + 2x + 10 = 0$$
$$x(x+5) + 2(x+5) = 0$$
$$(x+2)(x+5) = 0$$

So, the zeroes are $x = -2$ and $x = -5$. Let $\alpha = -2$ and $\beta = -5$.

Verification:

- Sum of zeroes $= \alpha + \beta = -2 + (-5) = -7$.
  Formula: $-\frac{b}{a} = -\frac{7}{1} = -7$. (Verified)
- Product of zeroes $= \alpha\beta = (-2)(-5) = 10$.
  Formula: $\frac{c}{a} = \frac{10}{1} = 10$. (Verified)

Example 2: Find a quadratic polynomial, the sum and product of whose zeroes are -3 and 2, respectively.

Solution:

Let the zeroes be α and β.

Given: $\alpha + \beta = -3$ and $\alpha\beta = 2$.

The required polynomial is given by:

$$p(x) = k\left[x^2 - (\alpha+\beta)x + \alpha\beta\right]$$

Substitute the values:

$$p(x) = k\left[x^2 - (-3)x + 2\right] = k(x^2 + 3x + 2)$$

Taking $k = 1$, the quadratic polynomial is $x^2 + 3x + 2$.

5 Practice Questions

Multiple Choice Questions (1 Mark Each)

1. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are:
   (a) Both positive
   (b) Both negative
   (c) One positive and one negative
   (d) Both equal

2. A quadratic polynomial whose zeroes are -3 and 4 is:
   (a) $x^2 - x + 12$
   (b) $x^2 + x + 12$
   (c) $\frac{x^2}{2} - \frac{x}{2} - 6$
   (d) $2x^2 + 2x - 24$

3. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is:
   (a) 10
   (b) -10
   (c) 5
   (d) -5

Short Answer Type Questions (2-3 Marks Each)

4. Find the zeroes of the polynomial $p(x) = 4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients.

5. Find a quadratic polynomial whose zeroes are $5 - 3\sqrt{2}$ and $5 + 3\sqrt{2}$.

6. If α and β are the zeroes of the polynomial $2x^2 - 4x + 5$, find the value of $\alpha^2 + \beta^2$.

7. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Long Answer Type Questions (4-5 Marks Each)

8. Find the zeroes of the quadratic polynomial $p(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$ and verify the relationship between the zeroes and the coefficients.

9. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 1$, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

10. Draw the graph of the polynomial $p(x) = x^2 - 4x + 3$. Find the zeroes of the polynomial from the graph and verify them algebraically.