Class 10 Mathematics

Comprehensive Study Material & Formula Sheet

Based on the Latest Syllabus Pattern

Contents

1 Real Numbers **2**
1.1 Fundamental Theorem of Arithmetic . . . 2
1.2 HCF and LCM Properties . . . 2
1.3 Rational and Irrational Numbers . . . 2

2 Polynomials **2**
2.1 Zeroes of a Polynomial . . . 2
2.2 Relationship between Zeroes and Coefficients . . . 2

3 Pair of Linear Equations in Two Variables **2**
3.1 Conditions for Consistency . . . 2

4 Quadratic Equations **3**
4.1 Quadratic Formula . . . 3
4.2 Nature of Roots . . . 3

5 Arithmetic Progressions (AP) **3**
5.1 General Term (n^{th} term) . . . 3
5.2 Sum of First n Terms (S_n) . . . 3

6 Coordinate Geometry **3**
6.1 Distance Formula . . . 3
6.2 Section Formula . . . 3
6.3 Mid-point Formula . . . 3

7 Introduction to Trigonometry **4**
7.1 Trigonometric Ratios . . . 4
7.2 Trigonometric Table for Standard Angles . . . 4
7.3 Trigonometric Identities . . . 4

8 Circles **4**

9 Areas Related to Circles **4**

10 Surface Areas and Volumes **5**

11 Statistics **5**

11.1 Mean ($\bar{x}$) 5
11.2 Mode 5
11.3 Median 5
11.4 Empirical Relationship 5

12 Probability **5**

1 Real Numbers

1.1 Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

1.2 HCF and LCM Properties

For any two positive integers a and b:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Note: This property is valid only for two numbers. It does not hold true for three or more numbers.

1.3 Rational and Irrational Numbers

- A number is **rational** if it can be written in the form p/q, where p and q are integers and $q \neq 0$.
- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer. This theorem is used to prove the irrationality of numbers like $\sqrt{2}, \sqrt{3}, \sqrt{5}$.

2 Polynomials

2.1 Zeroes of a Polynomial

A real number k is a zero of a polynomial $P(x)$ if $P(k) = 0$. Geometrically, the zeroes of a polynomial $P(x)$ are the x-coordinates of the points where the graph of $y = P(x)$ intersects the x-axis.

2.2 Relationship between Zeroes and Coefficients

For a quadratic polynomial $P(x) = ax^2 + bx + c$, let the zeroes be α and β:

$$\text{Sum of zeroes } (\alpha + \beta) = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$
$$\text{Product of zeroes } (\alpha\beta) = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

To form a quadratic polynomial when zeroes are given:

$$P(x) = k\left[x^2 - (\alpha + \beta)x + \alpha\beta\right] \quad \text{where } k \text{ is a non-zero real number.}$$

3 Pair of Linear Equations in Two Variables

Standard Form: $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

3.1 Conditions for Consistency

Ratio Comparison	Graphical Representation	Algebraic Interpretation	Consistency
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (Unique)	Consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	Consistent (Dependent)
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	Inconsistent

4 Quadratic Equations

Standard Form: $ax^2 + bx + c = 0$, where $a \neq 0$.

4.1 Quadratic Formula

The roots of a quadratic equation are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4.2 Nature of Roots

The term $D = b^2 - 4ac$ is called the **Discriminant**.

- If $D > 0$: Two distinct real roots.
- If $D = 0$: Two equal real roots (coincident roots).
- If $D < 0$: No real roots (imaginary roots).

5 Arithmetic Progressions (AP)

An arithmetic progression is a sequence of numbers in which the difference between any two consecutive terms is constant. Let a be the first term and d be the common difference.

5.1 General Term (n^{th} term)

$$a_n = a + (n-1)d$$

5.2 Sum of First n Terms (S_n)

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Alternatively, if the last term l is known:

$$S_n = \frac{n}{2}(a + l)$$

6 Coordinate Geometry

6.1 Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is:

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

6.2 Section Formula

The coordinates of point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

6.3 Mid-point Formula

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

7 Introduction to Trigonometry

7.1 Trigonometric Ratios

For a right-angled triangle, with reference to an acute angle θ:

- $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$
- $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$
- $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\sin\theta}{\cos\theta}$

7.2 Trigonometric Table for Standard Angles

Ratio / Angle (θ)	**0°**	**30°**	**45°**	**60°**	**90°**
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

7.3 Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \text{cosec}^2\theta$

8 Circles

- A tangent to a circle is perpendicular to the radius through the point of contact.
- The lengths of the two tangents drawn from an external point to a circle are equal.

9 Areas Related to Circles

Let r be the radius of the circle and θ be the angle of the sector in degrees.

- **Circumference of circle** $= 2\pi r$
- **Area of circle** $= \pi r^2$
- **Length of an arc of a sector** $= \frac{\theta}{360^\circ} \times 2\pi r$
- **Area of a sector** $= \frac{\theta}{360^\circ} \times \pi r^2$
- **Area of a segment** = Area of the corresponding sector − Area of the corresponding triangle.

10 Surface Areas and Volumes

Solid Shape	**Curved/Lateral Surface Area (CSA/LSA)**	**Total Surface Area (TSA)**	**Volume**
Cuboid (l, b, h)	$2h(l + b)$	$2(lb + bh + hl)$	$l \times b \times h$
Cube (side a)	$4a^2$	$6a^2$	a^3
Cylinder (r, h)	$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$
Cone (r, h, l)	πrl (where $l = \sqrt{r^2 + h^2}$)	$\pi r(r + l)$	$\frac{1}{3}\pi r^2 h$
Sphere (r)	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere (r)	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

11 Statistics

11.1 Mean ($\bar{x}$)

- **Direct Method:** $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$
- **Assumed Mean Method:** $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, where $d_i = x_i - a$

11.2 Mode

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where l = lower limit of modal class, h = class size, f_1 = frequency of modal class, f_0 = frequency of class preceding modal class, f_2 = frequency of class succeeding modal class.

11.3 Median

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where l = lower limit of median class, n = number of observations, cf = cumulative frequency of class preceding median class, f = frequency of median class, h = class size.

11.4 Empirical Relationship

$$3 \times \text{Median} = \text{Mode} + 2 \times \text{Mean}$$

12 Probability

The theoretical (classical) probability of an event E, written as $P(E)$, is defined as:

$$P(E) = \frac{\text{Number of outcomes favorable to } E}{\text{Total number of all possible outcomes of the experiment}}$$

- The probability of a sure event is 1.
- The probability of an impossible event is 0.
- For any event E, $0 \leq P(E) \leq 1$.
- $P(E) + P(\bar{E}) = 1$, where $\bar{E}$ represents 'not E'.