Class 10 Maths Practice Solutions

Based on the Latest Syllabus Pattern

Comprehensive step-by-step solutions for key concepts.

1. Chapter 1: Real Numbers

Question 1: Prove that $\sqrt{2}$ is an irrational number.

Solution: Let us assume, to the contrary, that $\sqrt{2}$ is a rational number. Then, there exist coprime integers p and q $(q \neq 0)$ such that:

$$\sqrt{2} = \frac{p}{q} \tag{1}$$

Squaring both sides, we get:

$$2 = \frac{p^2}{q^2} \implies 2q^2 = p^2 \quad \text{— (Equation 1)} \tag{2}$$

This means that 2 divides p^2. According to the fundamental theorem of arithmetic, if a prime number divides p^2, it must also divide p. Therefore, 2 divides p.

Let $p = 2c$ for some integer c. Substituting this value into Equation 1:

$$2q^2 = (2c)^2 \implies 2q^2 = 4c^2 \implies q^2 = 2c^2 \tag{3}$$

This means that 2 divides q^2, and consequently, 2 divides q.

Therefore, both p and q have at least 2 as a common factor. But this contradicts the fact that p and q are coprime (having no common factor other than 1). This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. Hence, we conclude that $\sqrt{2}$ is an irrational number.

Question 2: Find the HCF and LCM of 6, 72 and 120 using the prime factorization method.

Solution: First, we find the prime factorization of each number:

$$6 = 2^1 \times 3^1$$
$$72 = 2^3 \times 3^2$$
$$120 = 2^3 \times 3^1 \times 5^1$$

HCF (Highest Common Factor): The product of the smallest power of each common prime factor in the numbers. The common prime factors are 2 and 3. Smallest power of 2 is 2^1. Smallest power of 3 is 3^1.

$$\text{HCF}(6, 72, 120) = 2^1 \times 3^1 = 6$$

LCM (Least Common Multiple): The product of the greatest power of each prime factor involved in the numbers. The prime factors are $2, 3$, and 5. Greatest power of 2 is 2^3. Greatest power of 3 is 3^2. Greatest power of 5 is 5^1.

$$\text{LCM}(6, 72, 120) = 2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$$

2. Chapter 2: Polynomials

Question 1: Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$, and verify the relationship between the zeroes and the coefficients.

Solution: Let $p(x) = x^2 - 2x - 8$. To find the zeroes, we set $p(x) = 0$ and factorize the polynomial by splitting the middle term.

$$\begin{aligned} x^2 - 4x + 2x - 8 &= 0 \\ x(x-4) + 2(x-4) &= 0 \\ (x-4)(x+2) &= 0 \end{aligned}$$

So, the zeroes are $x = 4$ and $x = -2$. Let $\alpha = 4$ and $\beta = -2$.

Verification of Relationship: Comparing $x^2 - 2x - 8$ with the standard form $ax^2 + bx + c$, we get: $a = 1$, $b = -2$, $c = -8$.

1. **Sum of zeroes:**

$$\alpha + \beta = 4 + (-2) = 2$$

$$\text{Formula: } \frac{-b}{a} = \frac{-(-2)}{1} = 2$$

Since $2 = 2$, the sum of zeroes is verified.

2. **Product of zeroes:**

$$\alpha\beta = (4)(-2) = -8$$

$$\text{Formula: } \frac{c}{a} = \frac{-8}{1} = -8$$

Since $-8 = -8$, the product of zeroes is verified.

3. Chapter 4: Quadratic Equations

Question 1: Find the roots of the quadratic equation $2x^2 - 5x + 3 = 0$ using the quadratic formula.

Solution: The given equation is $2x^2 - 5x + 3 = 0$. Comparing it with $ax^2 + bx + c = 0$, we have: $a = 2$, $b = -5$, $c = 3$.

First, we calculate the discriminant (D):

$$\begin{aligned} D &= b^2 - 4ac \\ D &= (-5)^2 - 4(2)(3) \\ D &= 25 - 24 = 1 \end{aligned}$$

Since $D > 0$, the roots are real and distinct.

Using the quadratic formula:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{D}}{2a} \\ x &= \frac{-(-5) \pm \sqrt{1}}{2(2)} \\ x &= \frac{5 \pm 1}{4} \end{aligned}$$

This gives two possible values for x:

$$x = \frac{5+1}{4} = \frac{6}{4} = \frac{3}{2} \quad \text{and} \quad x = \frac{5-1}{4} = \frac{4}{4} = 1$$

Hence, the roots of the equation are $\frac{3}{2}$ and 1.

4. Chapter 8: Introduction to Trigonometry

Question 1: Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Solution: We know that $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{13}{12}$. Let the Hypotenuse be $13k$ and the Base be $12k$, where k is a positive integer.

Using Pythagoras Theorem in the right-angled triangle:

$$\begin{aligned}
(\text{Hypotenuse})^2 &= (\text{Perpendicular})^2 + (\text{Base})^2 \\
(13k)^2 &= (\text{Perpendicular})^2 + (12k)^2 \\
169k^2 &= (\text{Perpendicular})^2 + 144k^2 \\
(\text{Perpendicular})^2 &= 169k^2 - 144k^2 = 25k^2 \\
\text{Perpendicular} &= 5k
\end{aligned}$$

Now, we can find all other trigonometric ratios:

- $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{5k}{13k} = \frac{5}{13}$
- $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{12k}{13k} = \frac{12}{13}$ (Also, $\cos\theta = \frac{1}{\sec\theta}$)
- $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{5k}{12k} = \frac{5}{12}$
- $\text{cosec }\theta = \frac{1}{\sin\theta} = \frac{13}{5}$
- $\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$

Question 2: Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$
Solution: Let's take the Left Hand Side (LHS):

$$\text{LHS} = \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$$

Dividing the numerator and denominator by $\cos\theta$:

$$\text{LHS} = \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$\text{LHS} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$\text{LHS} = \frac{(\tan\theta + \sec\theta) - 1}{\tan\theta - \sec\theta + 1}$$

We know the identity: $\sec^2\theta - \tan^2\theta = 1$. Substituting this for the '1' in the numerator:

$$\text{LHS} = \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

Using $a^2 - b^2 = (a-b)(a+b)$:

$$\text{LHS} = \frac{(\sec\theta + \tan\theta) - [(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

Taking $(\sec\theta + \tan\theta)$ common from the numerator:

$$\text{LHS} = \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$\text{LHS} = \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

Canceling the common term $(1 - \sec\theta + \tan\theta)$:

$$\text{LHS} = \sec\theta + \tan\theta$$

Now, multiply and divide by $(\sec\theta - \tan\theta)$:

$$\text{LHS} = \frac{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\sec\theta - \tan\theta}$$

$$\text{LHS} = \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

Since $\sec^2\theta - \tan^2\theta = 1$:

$$\text{LHS} = \frac{1}{\sec\theta - \tan\theta} = \text{RHS}$$

Hence Proved.

5. Chapter 14: Statistics

Question 1: A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household.

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
Number of families	7	8	2	2	1

Find the mode of this data.

Solution: Here the maximum class frequency is 8, and the class corresponding to this frequency is $3 - 5$. So, the modal class is $3 - 5$.

- Lower limit (l) of modal class $= 3$
- Class size $(h) = 2$
- Frequency (f_1) of the modal class $= 8$
- Frequency (f_0) of class preceding the modal class $= 7$
- Frequency (f_2) of class succeeding the modal class $= 2$

The formula for Mode is:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Substituting the values:

$$\begin{aligned}\text{Mode} &= 3 + \left(\frac{8-7}{2(8)-7-2}\right) \times 2 \\ &= 3 + \left(\frac{1}{16-9}\right) \times 2 \\ &= 3 + \left(\frac{1}{7}\right) \times 2 \\ &= 3 + \frac{2}{7} \\ &= 3 + 0.286 \\ &= 3.286\end{aligned}$$

Therefore, the mode of the data is approximately 3.286.

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