Class 10 Maths NCERT-Based Solutions

Chapter 8: Introduction to Trigonometry

Comprehensive Step-by-Step Solutions for Board Exam Preparation

Key Formulas & Concepts

Before diving into the solutions, review these essential trigonometric concepts for a right-angled triangle $\triangle ABC$ (right-angled at B):

- **Sine** (sin): $\frac{\text{Opposite}}{\text{Hypotenuse}}$
- **Cosine** (cos): $\frac{\text{Adjacent}}{\text{Hypotenuse}}$
- **Tangent** (tan): $\frac{\text{Opposite}}{\text{Adjacent}} = \frac{\sin}{\cos}$
- **Pythagoras Theorem:** $Hypotenuse^2 = Base^2 + Perpendicular^2$
- **Identities:** $\sin^2\theta + \cos^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta, \quad 1 + \cot^2\theta = \text{cosec}^2\theta$

Exercise 8.1 Solutions

Question 1

In $\triangle ABC$, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine: (i) $\sin A, \cos A$ (ii) $\sin C, \cos C$

Solution:

1. **Find the Hypotenuse (AC):** By Pythagoras theorem in $\triangle ABC$:

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = (24)^2 + (7)^2$$
$$AC^2 = 576 + 49$$
$$AC^2 = 625$$
$$AC = \sqrt{625} = 25 \text{ cm}$$

2. **Calculate for Angle A:** For angle A, the opposite side is BC and the adjacent side is AB.

$$\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$
$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

3. **Calculate for Angle C:** For angle C, the opposite side is AB and the adjacent side is BC.

$$\sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$
$$\cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$$

Question 3

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Solution:

1. **Setup the triangle:** Let $\triangle ABC$ be a right-angled triangle, right-angled at B. Given $\sin A = \frac{3}{4} = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$. Let $BC = 3k$ and $AC = 4k$, where k is a positive constant.

2. **Find the adjacent side (AB):** Using Pythagoras theorem:

$$AC^2 = AB^2 + BC^2$$
$$(4k)^2 = AB^2 + (3k)^2$$
$$16k^2 = AB^2 + 9k^2$$
$$AB^2 = 16k^2 - 9k^2 = 7k^2$$
$$AB = \sqrt{7}k$$

3. **Calculate** $\cos A$ **and** $\tan A$**:**

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Exercise 8.2 Solutions

Question 1 (i)

Evaluate the following: $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

Solution:

1. **Recall standard trigonometric values:**

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$
$$\sin 30^\circ = \frac{1}{2}, \quad \cos 60^\circ = \frac{1}{2}$$

2. **Substitute values into the expression:**

$$\begin{aligned}\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{3}{4} + \frac{1}{4} \\ &= \frac{4}{4} \\ &= 1\end{aligned}$$

Question 1 (ii)

Evaluate: $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

Solution:

1. **Recall standard trigonometric values:**

$$\begin{aligned}\tan 45^\circ &= 1 \\ \cos 30^\circ &= \frac{\sqrt{3}}{2} \\ \sin 60^\circ &= \frac{\sqrt{3}}{2}\end{aligned}$$

2. **Substitute and solve:**

$$\begin{aligned}2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ &= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= 2(1) + \frac{3}{4} - \frac{3}{4} \\ &= 2 + 0 \\ &= 2\end{aligned}$$

Exercise 8.3 Solutions (Trigonometric Identities)

Question 4 (vii)

Prove the following identity: $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

Solution:

1. **Start with the Left Hand Side (LHS):**

$$\text{LHS}=\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}$$

2. **Factor out $\sin\theta$ from the numerator and $\cos\theta$ from the denominator:**

$$\text{LHS}=\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

3. **Use the identity $\sin^2\theta+\cos^2\theta=1$ in the numerator:** Replace 1 with $(\sin^2\theta+\cos^2\theta)$:

$$\begin{aligned}1-2\sin^2\theta&=(\sin^2\theta+\cos^2\theta)-2\sin^2\theta\\&=\cos^2\theta-\sin^2\theta\end{aligned}$$

4. **Use the same identity in the denominator:** Replace 1 with $(\sin^2\theta+\cos^2\theta)$:

$$\begin{aligned}2\cos^2\theta-1&=2\cos^2\theta-(\sin^2\theta+\cos^2\theta)\\&=\cos^2\theta-\sin^2\theta\end{aligned}$$

5. **Substitute back into the fraction:**

$$\text{LHS}=\frac{\sin\theta(\cos^2\theta-\sin^2\theta)}{\cos\theta(\cos^2\theta-\sin^2\theta)}$$

6. **Cancel the common terms and simplify:**

$$\text{LHS}=\frac{\sin\theta}{\cos\theta}=\tan\theta=\text{RHS}$$

Hence Proved.

Question 4 (viii)

Prove the following identity: $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

Solution:

1. **Expand the LHS using $(x+y)^2=x^2+y^2+2xy$:**

$$\begin{aligned}\text{LHS}=&(\sin^2 A+\operatorname{cosec}^2 A+2\sin A\operatorname{cosec} A)\\&+(\cos^2 A+\sec^2 A+2\cos A\sec A)\end{aligned}$$

2. **Use reciprocal identities ($\sin A \cdot$ cosec $A = 1$ and $\cos A \cdot \sec A = 1$):**

$$\begin{aligned} \text{LHS} &= \sin^2 A + \text{cosec}^2 A + 2(1) + \cos^2 A + \sec^2 A + 2(1) \\ &= \sin^2 A + \cos^2 A + \text{cosec}^2 A + \sec^2 A + 4 \end{aligned}$$

3. **Group terms and apply Pythagorean identities:** We know that $\sin^2 A + \cos^2 A = 1$.

$$\begin{aligned} \text{LHS} &= 1 + \text{cosec}^2 A + \sec^2 A + 4 \\ &= 5 + \text{cosec}^2 A + \sec^2 A \end{aligned}$$

4. **Apply remaining identities (cosec$^2 A = 1 + \cot^2 A$ and $\sec^2 A = 1 + \tan^2 A$):**

$$\begin{aligned} \text{LHS} &= 5 + (1 + \cot^2 A) + (1 + \tan^2 A) \\ &= 7 + \tan^2 A + \cot^2 A = \text{RHS} \end{aligned}$$

Hence Proved.